ABSTRACT: Although traditional per-phase equivalent circuit has been widely used in steady-state analysis and design of induction motors, it is not appropriate to predict dynamic performance of the motor. In order to understand and analyze vector control of induction motors, the dynamic model is necessary. Unfortunately, the dynamic model equations are complex and there are many different forms of the model depending on the choice of reference frame. It is the objective to explain various forms in a concise way to understand clearly. In addition, the fundamental dynamic mechanism of the motor in the synchronous frame is developed and the basic principles of vector control is discussed in general terms.

I. INTRODUCTION

The induction motor, which is the most widely used motor type in the industry, has been favored because of its good self-starting capability, simple and rugged structure, low cost and reliability, etc. Along with variable frequency AC inverters, induction motors are used in many adjustable speed applications which do not require fast dynamic response. The concept of vector control has opened up a new possibility that induction motors can be controlled to achieve dynamic performance as good as that of DC or brushless DC motors. In order to understand and analyze vector control, the dynamic model of the induction motor is necessary. It has been found that the dynamic model equations developed on a rotating reference frame is easier to describe the characteristics of induction motors. It is the objective of the article to derive and explain induction motor model in relatively simple terms by using the concept of space vectors and d-q variables. It will be shown that when we choose a synchronous reference frame in which rotor flux lies on the d-axis, dynamic equations of the induction motor is simplified and analogous to a DC motor.

Traditionally in analysis and design of induction motors, the “per-phase equivalent circuit” of induction motors shown in Fig. 1.1 has been widely used. In the circuit, $R_s$ ($R_r$) is the stator (rotor) resistance and $L_m$ is called the magnetizing inductance of the motor. Note that stator (rotor) inductance $L_s$ ($L_r$) is defined by

$$L_s = L_{ls} + L_m, \quad L_r = L_{lr} + L_m \tag{1.1}$$

where $L_{ls}$ ($L_{rs}$) is the stator (rotor) leakage inductance. Also note that in this equivalent circuit, all rotor parameters and variables are not actual quantities but are quantities referred to the stator [1]. Methods of determining circuit parameters from no-load test and locked rotor test are described in [2]. It is also known that induction motors do not rotate synchronously to the excitation frequency. At rated load, the speed of induction motors are slightly (about 2 - 7% slip in many cases) less than the synchronous speed. If the excitation frequency injected into the stator is $\omega_e$ and the actual speed converted into electrical frequency unit is $\omega_o$, slip $s$ is defined by

$$s = (\omega_o - \omega_e) / \omega_e = \omega_r / \omega_e \tag{1.2}$$

and $\omega_e$ is called the slip frequency which is the frequency of the actual rotor current. In the steady-state AC circuit, current and voltage phasors are used and they are denoted by the underline. In Fig. 1.1, power consumption in the stator is interpreted as $I_s^2 R_s$, while $I_r^2 R_r/s$ represents both power consumption in the rotor and the mechanical output (torque). By subtracting rotor loss $I_r^2 R_r$ from $I_s^2 R_s$, produced torque (mechanical power divided by the shaft speed) is given by

$$T = I_s^2 R_s (P/2) (1-s) / (s \omega_e) = I_r^2 R_r [ P / (2 \omega_e) ], \tag{1.3}$$

where $P$ is the number of poles. Although the per-phase equivalent circuit is useful in analyzing and predicting steady-state performance, it is not applicable to explain dynamic performance of the induction motor. In the next section, we will develop dynamic model of induction motors in general framework and introduce several equivalent circuits as special cases.
Throughout the article, all vectors are denoted as **boldface** and complex conjugates are denoted by \(^*\). Vectors on a rotating reference frame is followed by a superscript letter which designates the frame used as in \(Vs^s\) (Vs in stationary frame). The derivative operator is denoted by \(p\) while \(P\) is the number of poles. For notational convenience, let \(Y\) (scalar) or \(Y\) (vector) be the representative notation of any voltage, current or flux linkage variable. Real and Imaginary values of a space vector \(Y\) is denoted by \(\text{Re}(Y)\) and \(\text{Im}(Y)\), respectively. Zero vectors are denoted by \(\theta\) regardless of the reference frame used.

![Fig. 1.1 Conventional Per-phase Equivalent Circuit](image)

**II. DYNAMIC MODEL IN SPACE VECTOR FORM**

In an induction motor, the 3-phase stator windings are designed to produce sinusoidally distributed mmf in space along the airgap periphery. Assuming uniform airgap and neglecting the effects of slot harmonics, distribution of magnetic flux will also be sinusoidal. It is also assumed that the neutral connection of the machine is open so that phase voltages, currents and flux linkages are always balanced and there are no zero phase sequence component in the system. For such machines, the notation in terms of the space vector [3] is very useful. For 3 phase induction motors, the space vector \(Ys^s\) of the stator voltage, current and flux linkage is defined from its phase quantities by

\[
Ys^s = \frac{2}{3} (Y_a + \alpha Y_b + \alpha^2 Y_c),
\]  
(2.1)

where \(\alpha = exp(j 2\pi/3)\). The above transform is reversible and each phase quantities can be calculated from the space vector by,

\[
Y_a = \text{Re}(Ys^s), \quad I_b = \text{Re}(\alpha^2 Ys^s), \quad I_c = \text{Re}(\alpha Ys^s).
\]  
(2.2)

For a sinusoidal 3-phase quantity of constant rms value, the corresponding space vector is a constant-magnitude vector rotating at the frequency of the sinusoid with respect to the fixed (stationary) reference frame. Note that the space vector is at vector angle 0 when A-phase signal \((Ya)\) is at its sinusoidal peak value in steady-state. With space vector notation, voltage equations on the stator and rotor circuits of induction motors are,

\[
Vs^s = Rs Is^s + p \lambda s^s
\]  
(2.3)

\[
Vr' = Rr' Ir' + p \lambda r' = 0
\]  
(2.4)

It is very convenient to transform actual rotor variables \((Vr', Ir', \lambda r')\) from Eq. 2.4 on a rotor reference frame into a new variables \((Vr^s, Ir^s, \lambda r^s)\) on a stator reference frame as in the derivation of conventional steady-state equivalent circuit. Let the stator to rotor winding turn ratio be \(n\) and the angular position of the rotor be \(\theta\), and define

\[
Ir^s = \frac{1}{n} \exp(j \theta) \cdot Ir', \quad \lambda r^s = \frac{n}{\theta_0} \exp(j \theta) \cdot \lambda r'
\]  
(2.5)

Also, by defining referred rotor impedances as \(Rr = n^2 Rr', \text{etc.}\), we have

\[
Vs^s = Rs Is^s + p \lambda s^s
\]  
(2.6)

\[
0 = Rr Ir^s + (p - j\omega) \lambda r^s
\]  
(2.7)

where \(\omega_0 = p \theta_0\), is the speed of the motor in electrical frequency unit and

\[
\lambda s^s = Ls Is^s + Lm Ir^s
\]  
(2.8)
\[ \lambda_r^s = Lm I_s^s + Lr I_r^s \]  
(2.9)

The above 4 equations (Eq. 2.6 - 2.9) constitute a dynamic model of the induction motor on a stationary (stator) reference frame in space vector form. These model equations may be simplified by eliminating flux linkages as

\[ V_s^s = (R_s + L_s p) I_s^s + Lm p I_r^s \]  
(2.10)

\[ \theta = (R_r + L_r (p - j \omega_e)) I_r^s + Lm (p - j \omega_e) I_s^s. \]  
(2.11)

From Eqs. 2.10-2.11, the dynamic equivalent circuit model on a stationary reference frame can be drawn as in Fig. 2.1. For steady-state operation with excitation frequency \( \omega_e \), \( p \) in Eq. 2.10-2.11 may be replaced by \( j \omega_e \) and after some algebraic manipulation, we get

\[ V_s^s = (R_s + j \omega_e L_s) I_s^s + Lm p I_r^s \]  
(2.12)

\[ \theta = (R_r/ s + j \omega_e L_r) I_r^s + j \omega_e Lm I_s^s. \]  
(2.13)

which exactly describes the conventional steady-state equivalent circuit of Fig. 1.1.

Now, the previous procedure can be generalized so that the dynamic model is described on an arbitrary reference frame rotating at a speed \( \omega_a \), where Eq. 2.6 -2.13 is a special case with \( \omega_a = 0 \). To do that, define the new space vector on the arbitrary frame as

\[ Y^a = \exp(-j \theta_a) Y^s \]  
(2.14)

and reconstruct all the model equations in terms of the new space vectors. In the arbitrary reference frame, Eqs. 2.6-2.7 are modified to

\[ V_s^a = (R_s + L_s p) I_s^a + Lm p I_r^a + j \omega_a \lambda_s^a \]  
(2.15)

\[ \theta = (R_r + L_r p) I_r^a + Lm p I_s^a + j (\omega_a - \omega_e) \lambda_r^a, \]  
(2.16)

with new flux linkage equations defined by,

\[ \lambda_s^a = L_s I_s^a + Lm I_r^a \]  
(2.17)

\[ \lambda_r^a = Lm I_s^a + Lr I_r^a \]  
(2.18)

As before, by substituting Eqs. 2.16-2.17 into Eqs. 2.14-2.15, we have

\[ V_s^a = (R_s + L_s (p + j \omega_a)) I_s^a + Lm (p + j \omega_e) I_r^a \]  
(2.19)

\[ \theta = (R_r + L_r (p + j \omega_e - j \omega_a)) I_r^a + Lm (p + j \omega_e - j \omega_a) I_s^a \]  
(2.20)

where eliminated flux linkage variables are eliminated.

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Fig. 2.1 Dynamic Equivalent Circuit on a Stationary Reference Frame

The generalized equivalent circuit on an arbitrarily rotating frame based on Eq. 2.19-2.20 is shown in Fig. 2.2. Now, depending on a specific choice of \( \omega_a \), many forms of dynamic equivalent circuit can be established. Among them, the synchronous frame form can be obtained by choosing \( \omega_a = \omega_e \). This form is very useful in describing the concept of vector control of induction motors as well as of PM synchronous motors because at this rotating frame, space
vector is not rotating, but fixed and have a constant magnitude in steady-state. Since space vectors in the synchronous frame will frequently be used, they are denoted without any superscript indicating the type of frame. Another possible reference frame used in vector control is the rotor reference frame by choosing \( \omega_c = \omega_b \) which is, in fact, the reverse step of Eq. 2.5 with \( n=1 \).

![Dynamic Equivalent Circuit](image)

**Fig. 2.2 Dynamic Equivalent Circuit on an Arbitrary Reference Frame Rotating at \( \omega_a \).**

### III. D-Q EQUIVALENT CIRCUIT

In many cases, analysis of induction motors with space vector model is complicated due to the fact that we have to deal with variables of complex numbers. For any space vector \( Y \), define two real quantities \( S_q \) and \( S_d \) as,

\[
S = S_q - j S_d
\]

In other words, \( S_q = \text{Re} (S) \) and \( S_d = -\text{Im} (S) \). Fig. 3.1 illustrates the relationship between d-q axis and complex plane on a rotating frame with respect to stationary a-b-c frame. Note that d- and q-axes are defined on a rotating reference frame at the speed of \( \omega_a = \rho \theta_a \) with respect to fixed a-b-c frame.

![D-q Axis Definition](image)

**Fig. 3.1 Definition of d-axis and q-axis on an arbitrary reference frame**

With the above definition, Eq. 2.19-2.20 can be translated into the following 4 equations of real variables expressed in a matrix form.

\[
\begin{bmatrix}
V_{qs}^a \\
V_{ds}^a \\
0 \\
0
\end{bmatrix} =
\begin{bmatrix}
Rs + Ls p & \omega_a Ls & Lm p & \omega_a Lm \\
-\omega_a Ls & Rs + Ls p & -\omega_a Lm & Lm p \\
0 & Lm p & (\omega_a - \omega_b) Lm & Rr + Lr p \\
0 & -(\omega_a - \omega_b) Lm & Lm p & -(\omega_a - \omega_b) Lr + Lr p
\end{bmatrix}
\begin{bmatrix}
I_{qs}^a \\
I_{ds}^a \\
I_{qa}^a \\
I_{da}^a
\end{bmatrix}
\]

\[
(3.2)
\]
For future reference, the above matrix equation simplified for popular reference frames in analysis and design of vector control will be introduced. For stationary reference frame, by substituting $\omega_a = 0$, the above equation is reduced to

$$
\begin{bmatrix}
V_qss \\
V_dss \\
0 \\
0
\end{bmatrix}
=
\begin{bmatrix}
Rs + Ls p & 0 & Lm p & 0 \\
0 & Rs + Ls p & Lm p & Lm p \\
\omega_k Lm & -\omega_k Lm & Rr + Lr p & -\omega_k Lr \\
0 & \omega_k Lm & Lm p & \omega_k Lr & Rr + Lr p
\end{bmatrix}
\begin{bmatrix}
I_{qss} \\
I_{dss} \\
I_{qrs} \\
I_{qrs}
\end{bmatrix}
(3.3)
$$

Some implementation of vector drive includes calculation in rotor reference frame (frame is attached to the rotor rotating at $\omega_o$). In this case, we can substitute all $\omega_a$ in Eq. 3.2 by $\omega_o$, which makes simplified rotor voltage equations. Moreover, for synchronous frame, we have

$$
\begin{bmatrix}
V_q s \\
V_d s \\
0 \\
0
\end{bmatrix}
=
\begin{bmatrix}
Rs + Ls p & \omega_k Ls & Lm p & \omega_k Lm \\
-\omega_k Ls & Rs + Ls p & -\omega_k Lm & Lm p \\
0 & \omega_k Lm & Rr + Lr p & \omega_k Lr \\
0 & -\omega_k Lm & Lm p & -\omega_k Lr & Rr + Lr p
\end{bmatrix}
\begin{bmatrix}
I_{q s} \\
I_{d s} \\
I_{q r} \\
I_{d r}
\end{bmatrix}
(3.4)
$$

As mentioned before, each variable (voltage, current or flux linkage) in the synchronous frame is stationary and fixed to a constant magnitude in steady-state. Based on Eq. 3.4, dynamic d-q equivalent circuit is shown in Fig. 3.2.

For dynamic simulation of induction motors, Eq. 3.3 or Eq. 3.4 may be used. In this case, one may prefer to use the standard form of differential equation as

$$
p X = A X + B U.
(3.5)
$$

For Eq. 3.4, matrix quantities on the above equation are as follows.
\[
X = \begin{bmatrix} I_{qs} \\ I_{ds} \\ I_{qr} \\ I_{dr} \end{bmatrix}, \quad U = \begin{bmatrix} V_{qs} \\ V_{ds} \\ V_{qr} \\ V_{dr} \end{bmatrix}, \quad B = \frac{1}{\Delta} \begin{bmatrix} Lr & 0 & -Lm & 0 \\ 0 & Lr & 0 & -Lm \\ -Lm & 0 & Ls & 0 \\ 0 & -Lm & 0 & Ls \end{bmatrix}
\]

and
\[
A = \frac{1}{\Delta} \begin{bmatrix} R_s Lr & \omega_s Lr - \omega_q Lm^2 & -Rr Lm & \omega_q Lm Lr \\ \omega_q Lm^2 - \omega_s Ls Lr & R_s Lr & -\omega_s Lr Lm & -Rr Lm \\ -R_s Lm & -\omega_s Ls Lm & Rr Ls & \omega_q Ls Lr - \omega_q Lm^2 \\ \omega_q Ls Lm & -R_s Lm & \omega_q Lm Lr - \omega_q Ls Lr & Rr Ls \end{bmatrix}
\]

In the above equation, \( \Delta = L_s L_r - L_m^2 \). Although both Eq. 3.4 and Eq. 3.5 are frequently used to describe the induction motor on a synchronous frame, we need another set of equations that include flux linkage variables to explain the concept of vector control. By translating Eq. 2.15 - 2.18 in d-q coordinate on a synchronous frame, we have the following 8 equations. Both stator and rotor voltage equations are,

\[
V_{qs} = R_s I_{qs} + p \lambda_{qs} + \omega_s \lambda_{ds}
\]
\[
V_{ds} = R_s I_{ds} + p \lambda_{ds} - \omega_s \lambda_{qs}
\]
\[
0 = R_r I_{qr} + p \lambda_{qr} + \omega_r \lambda_{dr}
\]
\[
0 = R_r I_{dr} + p \lambda_{dr} - \omega_r \lambda_{qr}
\]

where flux linkage variables are defined by

\[
\lambda_{qs} = L_s I_{qs} + L_m I_{qr}
\]
\[
\lambda_{ds} = L_s I_{ds} + L_m I_{dr}
\]
\[
\lambda_{qr} = L_m I_{qs} + L_r I_{qr}
\]
\[
\lambda_{dr} = L_m I_{ds} + L_r I_{dr}
\]

It will be shown in the next section that the above equation are very useful in explaining the dynamic structure of the motor and the concept of vector control.

When induction motors are controlled by a vector drive, control computation is often done in the synchronous frame. Since actual stator variables either to be generated or to be measured are all in stationary a-b-c frame, frame transform should be executed in the control. The most popular transform is between stationary a-b-c frame quantities to synchronously rotating d-q quantities. Combining Eq. 2.1, Eq. 2.14, and Eq. 3.1, we have

\[
S_{qs} = (2/3) R_e \{ \exp(-j\theta a) (Sa + \alpha Sb + \alpha^2 Sc) \} \quad (3.16)
\]
\[
S_{ds} = - (2/3) I_m \{ \exp(-j\theta a) (Sa + \alpha Sb + \alpha^2 Sc) \} \quad (3.17)
\]

Or in a simpler form,

\[
\begin{bmatrix} Y_q \\ Y_d \\ 0 \end{bmatrix} = (2/3) \begin{bmatrix} \cos \theta & \cos(\theta - 2\pi/3) & \cos(\theta + 2\pi/3) \\ \sin \theta & \sin(\theta - 2\pi/3) & \sin(\theta + 2\pi/3) \\ 0.5 & 0.5 & 0.5 \end{bmatrix} \begin{bmatrix} Y_a \\ Y_b \\ Y_c \end{bmatrix} \quad (3.18)
\]

and its inverse transform is given by

\[
\begin{bmatrix} Y_a \\ Y_b \\ Y_c \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta & 1 \\ \cos(\theta - 2\pi/3) & \sin(\theta - 2\pi/3) & 1 \\ \cos(\theta + 2\pi/3) & \sin(\theta + 2\pi/3) & 1 \end{bmatrix} \begin{bmatrix} Y_q \\ Y_d \\ 0 \end{bmatrix} \quad (3.19)
\]
In vector control drives, Eq. 3.18 is frequently used to convert measured currents and voltages to d-q quantities while Eq. 3.19 may be used to feed command signals to the amplifier. In many modern drives, Eq. 3.19 can be accomplished in a slightly different mechanism such as the space vector modulation [6].

Regardless of reference frame, instantaneous input power can be expressed, in terms of space vectors, by

\[
P_i = \frac{3}{2} \Re(V_s I_s^\theta),
\]

or in terms of d-q variables as

\[
P_i = \frac{3}{2} [ V_{ds} I_{ds} + V_{qs} I_{qs} ].
\]

The reactive power \( Q_i \) can also be defined as

\[
Q_i = \frac{3}{2} \Im(V_s I_s^\theta),
\]

or in terms of d-q variables as

\[
Q_i = \frac{3}{2} [ V_{qs} I_{ds} - V_{ds} I_{qs} ].
\]

This reactive power can be used in some parameter adaptation methods which automatically corrects the rotor time constant parameter (\( T_r \)) during steady-state operation.

Now, one simple way of obtaining the output torque is to consider the power associated with speed voltage term on Fig. 2.1 as

\[
P_o = \frac{3}{2} \omega, \Im(\lambda_r I_r^\theta).
\]

Since torque is the above power divided by the rotor speed,

\[
T_o = \frac{3}{4} \omega \Im(\lambda_r I_r^\theta),
\]

where \( P \) is the number of poles. In terms of d-q variables, Eq. 3.25 is

\[
T_o = \frac{3}{4} \omega \Im(\lambda_{qr} I_{dr} - \lambda_{dr} I_{qr}).
\]

Although the torque expression on the above is derived from stationary reference frame, it is true for any other reference frames. Many other forms of torque equations are possible [4]. For example, by substituting flux linkage relation of Eq. 2.9 into Eq. 3.25, we have

\[
T_o = \frac{3}{4} \omega \Im(\lambda_{qr} I_{dr} - \lambda_{dr} I_{qr}).
\]

Again, by using Eq. 2.9, we can eliminate \( I_r \) on Eq. 3.27 to get

\[
T_o = \frac{3}{4} \omega \Im(\lambda_{qr} I_{dr} - \lambda_{dr} I_{qr}).
\]

It will be shown later that Eqs. 3.29 - 3.30 are particularly important in vector control because output torque is expressed in terms of stator current and rotor flux linkage.

IV. PRINCIPLES OF VECTOR CONTROL

So far, we have not paid attention to the alignment of the rotating reference frame with respect to the physical coordinate. Noting in Eq. 3.28 that torque is directly proportional to \( I_{qs} \) if \( \lambda_{qr} = 0 \), one can choose the rotating d-axis to be the angle of the rotor flux linkage. In fact, this choice offers a lot of advantages of simplifying control and analysis of the motor. Other choices frequently used in direct vector control are stator flux linkage frame (d-axis is aligned to the stator flux linkage) and airgap flux linkage frame, which will be discussed briefly at the end of the section.

When the motor is driven from an ideal current source amplifier, Eq. 3.8-3.9 are automatically satisfied by the source and can be neglected in the analysis. This is practically true on many PWM voltage amplifiers which
have high bandwidth closed-loop current control. Assume that the rotor flux always coincides with the rotating d-axis frame, i.e.,

$$\lambda_r = -j \lambda_{dr}$$  \hspace{1cm} (4.1)

Then we have,

$$\lambda_{qr} = 0, \quad p \lambda_{qr} = 0.$$  \hspace{1cm} (4.2)

Applying the above conditions to Eq. 3.10-3.11, we have

$$Rr I_{qr} + \omega_r \lambda_{dr} = 0$$  \hspace{1cm} (4.3)

$$Rr I_{dr} + p \lambda_{dr} = 0$$  \hspace{1cm} (4.4)

Next, substitution of these relations into Eq. 3.14-3.15 yields

$$I_{qr} = -(L_m / L_r) I_{qs}$$  \hspace{1cm} (4.5)

$$I_{dr} = (\lambda_{dr} - L_m I_{ds}) / L_r$$  \hspace{1cm} (4.6)

Now, by defining the rotor time constant \(\tau_r\), a very, very important constant in induction motor dynamics as,

$$\tau_r = L_r / R_r,$$  \hspace{1cm} (4.7)

we have the following two equations.

$$\omega_{\lambda} = (L_m / \tau_r) (I_{qs} / \lambda_{dr})$$  \hspace{1cm} (4.8)

$$p \lambda_{dr} = (1 / \tau_r) (- \lambda_{dr} + L_m I_{ds})$$  \hspace{1cm} (4.9)

In the mean time, torque expression of Eq. 3.28 is reduced to

$$T = (3/4) P (L_m / L_r) \lambda_{dr} I_{qs}$$  \hspace{1cm} (4.10)

Based on Eqs. 4.8-4.10, we can draw a block diagram as in Fig. 4.1 of induction motor dynamics when rotor flux field oriented condition (Eq. 4.1) is imposed.

![Fig. 4.1 Block Diagram of Induction Motor Dynamics](image-url)

From the block diagram of Fig. 4.1, we can observe that the output torque is directly proportional to the q-axis stator current without dynamics while it is subject to a first order dynamics with time constant \(\tau_r\) from the d-axis stator current. In addition, rotor flux linkage is not affected by the change in \(I_{qs}\) (decoupled). We can also see that \(I_{dr}\) exists only when \(\lambda_{dr}\) (Eq. 4.4) changes due to the change in \(I_{ds}\) (Eq. 4.6). In steady-state, the magnitude of the rotor flux is

$$\lambda_{dr} = L_m I_{ds}$$  \hspace{1cm} (4.11)
This situation is analogous to the control characteristics of separately excited DC motors where torque is proportional to armature current, while field flux which has a long time lag due to high inductance of field circuit. Note in the above block diagram that determination of d-axis and q-axis stator currents from 3-phase input currents are based on the rotor flux angle. Inside the motor, rotor flux angle is determined by the angular position of the rotor plus integrated slip frequency which is given by Eq. 4.8.

In vector control of induction motors, the accuracy of rotor flux angle is critical in control because calculation of currents (Ids, Iqs) in the synchronous frame is determined by the rotor flux angle. Basically, there are two methods of determining rotor flux angle in vector control. One method, called Indirect Vector Control (IVC) calculates θs from

\[ ω_s^* = \frac{Lm}{τ} \left( Iq_s^* / λ_d^* \right) \]  
\[ θe^* = θo + \int θ_s^* \, dt \]

where quantities that are commanded or estimated in drive control are denoted by asterisk (*). Since this method relies on knowledge of motor parameters such as \( Lm \) and \( τ \), and the real values of which may be changing as operating conditions change, consideration should be given in design to the effects of parameter variations. Another method, called Direct Vector Control (DVC) determines \( θe^* \) either from the measurement of airgap flux, or from terminal voltages and currents. In the latter case, angle and magnitude of the rotor flux may be calculated by

\[ λ_s = \int (V_s - R_s I_s) \, dt. \]  
\[ λ_r = \frac{Lm}{Lr} (λ_s - L_o I_s). \]

where \( L_o = Ls - Lm^2/Lr \). Although DVC may be relatively insensitive to the variations (depending on the actual implementation) of rotor parameters, performance of DVC may be sluggish at low speed operation due to inaccurate knowledge on the stator resistance, integration drift, etc.

In the above discussions, we chose the reference frame d-axis to coincide with rotor flux linkage. This is called “rotor flux orientation.” Sometimes, “stator flux orientation” or “air-gap flux orientation” can be used. The airgap flux space vector is defined by

\[ λ_m = Lm I_s + Lm I_r. \]

All three orientation methods use the synchronous reference frame with slight differences in the choice of the reference vector. In any of the above cases, the description up to the previous section for the synchronous frame are applicable. In DVC, stator flux orientation may be used when flux linkage is calculated from terminal voltages and currents, while the airgap flux orientation may be preferred when actual airgap flux sensor is used for direct measurement on the motor. Since we do not have a nice decoupled torque relation shown in Fig.4.1 on both stator and airgap flux orientation methods, additional decoupling compensation should be applied for vector control in either stator or airgap flux orientation. Refer to [7] for further details.

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