

# DYNAMIC MODEL OF PM SYNCHRONOUS MOTORS

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**ABSTRACT:** *In a permanent magnet synchronous motor where inductances vary as a function of rotor angle, the 2 phase (d-q) equivalent circuit model is commonly used for simplicity and intuition. In this article, a two phase model for a PM synchronous motor is derived and the properties of the circuits and variables are discussed in relation to the physical 3 phase entities. Moreover, the paper suggests methods of obtaining complete model parameters from simple laboratory tests. Due to the lack of developed procedures in the past, obtaining model parameters were very difficult and uncertain, because some model parameters are not directly measurable and vary depending on the operating conditions. Formulation is mainly for interior permanent magnet synchronous motors but can also be applied to surface permanent magnet motors.*

## I. INTRODUCTION

PM synchronous motors are now very popular in a wide variety of industrial applications. A large majority of them are constructed with the permanent magnets mounted on the periphery of the rotor core. Following [1], we will call them as the Surface Permanent Magnet (SPM) synchronous motors. When permanent magnets are buried inside the rotor core rather than bonded on the rotor surface, the motor not only provides mechanical ruggedness but also opens a possibility of increasing its torque capability. By designing a rotor magnetic circuit such that the inductance varies as a function of rotor angle, the reluctance torque can be produced in addition to the mutual reaction torque of synchronous motors. This class of Interior PM (IPM) synchronous motors can be considered as the reluctance synchronous motor and the PM synchronous motor combined in one unit. It is now very popular in industrial and military applications by providing high power density and high efficiency compared to other types of motors.

Conventionally, a 2-phase equivalent circuit model (d-q model) [2] has been used to analyze reluctance synchronous machines. The theory is now applied in analysis of other types of motors [3-7] including PM synchronous motors, induction motors etc. In Section II, an equivalent 2-phase circuit model of a 3-phase IPM machines is derived in order to clarify the concept of the transformation and the relation between 3-phase quantities and their equivalent 2-phase quantities. Although the above equivalent circuit is very popular, discussions on obtaining parameters of the equivalent circuit for a given motor are rarely found. The main objective of the article is to establish a method to obtain 2-phase circuit parameters from physically measured data. Throughout the article, the following assumptions are made:

- (1) Stator windings produce sinusoidal mmf distribution. Space harmonics in the air-gaps are neglected.
- (2) Air-gap reluctance has a constant component as well as a sinusoidally varying component.
- (3) Balanced 3 phase supply voltage is considered.
- (4) Although magnetic saturation is considered, eddy current and hysteresis effects are neglected.

In addition, presence of damper windings are not considered here because they are not used in PM synchronous machines in general. A model with short-circuited damper windings may be used to analyze eddy current effects. Nomenclature on this article is listed in the following. More definitions may appear as appropriate during discussion. For notational convenience, units on all angles are in degrees.

- P: number of poles of the motor.  
I<sub>a</sub>, I<sub>b</sub>, I<sub>c</sub>: phase a, b, c instantaneous stator current.  
V<sub>a</sub>, V<sub>b</sub>, V<sub>c</sub>: phase a, b, c instantaneous stator voltage.  
I<sub>d</sub>, I<sub>q</sub>: d- and q- axis components of stator current

$V_d, V_q$ :	d and q- axis components of stator phase voltage
$R_s$ :	stator resistance
$p$ :	$d/dt$
$L_d, L_q$ :	d- and q- axis stator self inductance
$L_s$ :	Average inductance. $L_s = (1/2) (L_q + L_d)$
$L_x$ :	Inductance fluctuation. $L_x = (1/2) (L_q - L_d)$
$\lambda_m$ :	peak flux linkage due to permanent magnet.
$\theta$ :	electrical angle between a-axis and q-axis in degrees. See Fig. 2.1.
$\omega$ :	$\omega = p \dot{\theta}$ , angular velocity of rotation (in electrical rad/Sec.)

## II. DERIVATION OF A PM SYNCHRONOUS MOTOR MODEL

Fig. 2.1 illustrates a conceptual cross-sectional view of a 3-phase, 2-pole IPM synchronous motor along with two reference frames. To illustrate the inductance difference ( $L_q > L_d$ ), rotor is drawn with saliency although actual rotor structure is more likely a cylinder. The stator reference axis for the a-phase is chosen to the direction of maximum mmf when a positive a-phase current is supplied at its maximum level. Reference axis for b- and c- stator frame are chosen  $120^\circ$  and  $240^\circ$  (electrical angle) ahead of the a-axis, respectively. Following the convention of choosing the rotor reference frame, the direction of permanent magnet flux is chosen as the d-axis, while the q-axis is  $90$  degrees ahead of the d-axis. The angle of the rotor q-axis with respect to the stator a-axis is defined as  $\theta$ . Note that as the machine turns, the d-q reference frame is rotating at a speed of  $\omega = d\theta/dt$ , while the stator a-,b-,c- axes are fixed in space. We will find out later that the choice of this rotating frame greatly simplifies the dynamic equations of the model.

The electrical dynamic equation in terms of phase variables can be written as

$$V_a = R_s I_a + p \lambda_a \quad (2.1)$$

$$V_b = R_s I_b + p \lambda_b \quad (2.2)$$

$$V_c = R_s I_c + p \lambda_c \quad (2.3)$$

while the flux linkage equations are

$$\lambda_a = L_{aa} I_a + L_{ab} I_b + L_{ac} I_c + \lambda_{ma} \quad (2.4)$$

$$\lambda_b = L_{ab} I_a + L_{bb} I_b + L_{bc} I_c + \lambda_{mb} \quad (2.5)$$

$$\lambda_c = L_{ac} I_a + L_{bc} I_b + L_{cc} I_c + \lambda_{mc}, \quad (2.6)$$

considering symmetry of mutual inductances such as  $L_{ab}=L_{ba}$ . Note that in the above equations, inductances are functions of the angle  $\theta$ . Since stator self inductances are maximum when the rotor q-axis is aligned with the phase, while mutual inductances are maximum when the rotor q-axis is in the midway between two phases. Also, note that the effects of saliency appeared in stator self and mutual inductances are indicated by the term  $2\theta$ .

$$L_{aa} = L_{so} + L_{sl} + L_x \cos(2\theta) \quad (2.7)$$

$$L_{bb} = L_{so} + L_{sl} + L_x \cos(2\theta + 120) \quad (2.8)$$

$$L_{cc} = L_{so} + L_{sl} + L_x \cos(2\theta - 120) \quad (2.9)$$

$$L_{ab} = -(1/2)L_{so} + L_x \cos(2\theta - 120) \quad (2.10)$$

$$L_{bc} = -(1/2)L_{so} + L_x \cos(2\theta) \quad (2.11)$$

$$L_{ac} = -(1/2)L_{so} + L_x \cos(2\theta + 120) \quad (2.12)$$

For mutual inductances in the above equations, the coefficient  $-(1/2)$  comes due to the fact that stator phases are displaced by  $120^\circ$ , and  $\cos(120) = -(1/2)$ . Meanwhile, flux-linkages at the stator windings due to the permanent magnet are

$$\lambda_{ma} = \lambda_m \cos \theta \quad (2.13)$$

$$\lambda_{mb} = \lambda_m \cos (\theta - 120) \quad (2.14)$$

$$\lambda_{mc} = \lambda_m \cos (\theta + 120) \quad (2.15)$$

For this model, input power  $P_i$  can be represented as

$$P_i = V_a I_a + V_b I_b + V_c I_c \quad (2.16)$$

Unfortunately, the output power  $P_o$  and the output torque  $T = (P/2) P_o/\omega$  cannot simply be derived in this 3-phase model. The torque can be expressed as

$$T = (P/6) (L_q - L_d) [ \{ (I_a^2 - 0.5 I_b^2 - 0.5 I_c^2 - I_a I_b - I_a I_c + 2 I_b I_c) \sin 2\theta + (\sqrt{3}/2) (I_b^2 + I_c^2 - 2 I_a I_b + 2 I_a I_c) \cos 2\theta \} + \lambda_m \{ (I_a - 0.5 I_b - 0.5 I_c) \cos \theta + (\sqrt{3}/2) (I_b - I_c) \sin \theta \} ]. \quad (2.17)$$

Refer to [6] for detailed derivation by using energy method.

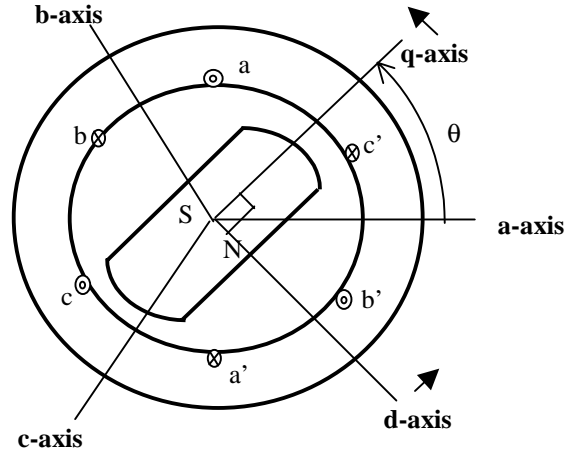


Fig. 2.1 PM Synchronous Motor

Now, let  $S$  represent any of the variables (current, voltage, and flux linkage) to be transformed from the a-b-c frame to d-q frame. The transformation in matrix form is given by

$$\begin{bmatrix} S_q \\ S_d \\ S_o \end{bmatrix} = (2/3) \begin{bmatrix} \cos \theta & \cos (\theta - 120) & \cos (\theta + 120) \\ \sin \theta & \sin (\theta - 120) & \sin (\theta + 120) \\ 0.5 & 0.5 & 0.5 \end{bmatrix} \begin{bmatrix} S_a \\ S_b \\ S_c \end{bmatrix} \quad (2.18)$$

Here  $S_o$  component is called the zero sequence component, and under balanced 3-phase system this component is always zero. Since it is a linear transformation, its inverse transformation exists and is

$$\begin{bmatrix} S_a \\ S_b \\ S_c \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta & 1 \\ \cos (\theta - 120) & \sin (\theta - 120) & 1 \\ \cos (\theta + 120) & \sin (\theta + 120) & 1 \end{bmatrix} \begin{bmatrix} S_q \\ S_d \\ S_o \end{bmatrix} \quad (2.19)$$

Now, by applying the transform of Eq. 2.18 to voltages, flux-linkages and currents of Eqs.2.1-2.6, we get a set of simple equations as

$$V_q = R_s I_q + p \lambda_q + \omega \lambda_d \quad (2.20)$$

$$V_d = R_s I_d + p \lambda_d - \omega \lambda_q, \quad (2.21)$$

where

$$\lambda_q = L_q I_q \quad (2.22)$$

$$\lambda_d = L_d I_d + \lambda_m. \quad (2.23)$$

Here,  $L_q$  and  $L_d$  are called d- and q-axis synchronous inductances, respectively, and are defined as

$$L_q = (3/2) (L_{so} + L_x) + L_{sl} \quad (2.24)$$

$$L_d = (3/2) (L_{so} - L_x) + L_{sl}. \quad (2.25)$$

As noticed in the above equation, synchronous inductances are effective inductances under balanced 3 phase conditions. Each synchronous inductance is made up of self inductance (which includes leakage inductance) and contributions from other 2 phase currents. Now, a more convenient equation may result by eliminating the flux-linkage terms from Eqs.2.20-2.21 as

$$V_q = (R_s + L_q p) I_q + \omega L_d I_d + \omega \lambda_m \quad (2.26)$$

$$V_d = (R_s + L_d p) I_d - \omega L_q I_q \quad (2.27)$$

Fig. 2.1 shows a dynamic equivalent circuit of an IPM synchronous machine based on Eqs.2.26-2.27. Note that in practice, magnetic circuits are subject to saturation as current increases. Especially, when  $I_q$  is increased, the value of  $L_q$  is decreased and  $\lambda_m$  and  $L_d$  is subject to armature reaction. Since  $I_d$  is maintained to zero or negative value (demagnetizing) in most operating conditions, saturation of  $L_d$  rarely occurs.

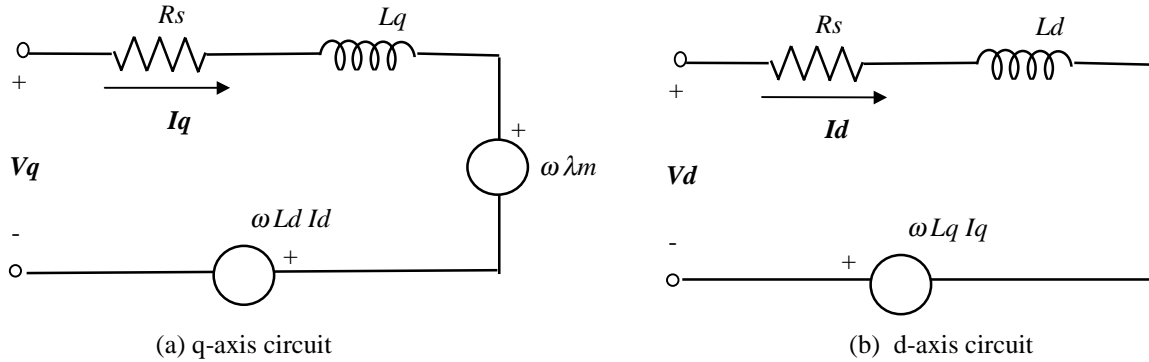


Fig. 2.1 Equivalent Circuit of a PM Synchronous Motor

For this model, instantaneous power can be derived from Eq. 2.16 via transformation as

$$P_i = (3/2) \{ V_q I_q + V_d I_d \}, \quad (2.28)$$

neglecting the zero sequence quantities. The output power can be obtained by replacing  $V_q$  and  $V_d$  by the associated speed voltages as

$$P_o = (3/2) \{ -\omega \lambda_q I_d + \omega \lambda_d I_q \} \quad (2.29)$$

The produced torque  $T$ , which is power divided by mechanical speed can be represented as

$$T = (3/2) (P/2) (\lambda_m I_q + (L_d - L_q) I_q I_d). \quad (2.30)$$

It is apparent from the above equation that the produced torque is composed of two distinct mechanisms. The first term corresponds to "the mutual reaction torque" occurring between  $I_q$  and the permanent magnet, while the second term corresponds to "the reluctance torque" due to the differences in d-axis and q-axis reluctance (or inductance). Note that in order to produce additive reluctance torque,  $I_d$  must be negative since  $L_q > L_d$ .

In order to discuss about the relation between the original 3-phase system and the 2-phase equivalent system, consider the transformation of Eq. 2.18-2.19. In fact, this reversible linear transformation can be interpreted as a combination of two transformations. First, let  $\alpha$ - $\beta$  axis be a stationary frame so that  $\alpha$ -axis and  $\beta$ -axis coincide with q- and d-axis, respectively when the angle  $\theta$  is zero. The transformation from 3-axis variables to 2-axis variables is

$$\begin{bmatrix} S\alpha \\ S\beta \\ S\sigma \end{bmatrix} = (2/3) \begin{bmatrix} 1 & \cos(120^\circ) & \cos(120^\circ) \\ 0 & -\sin(120^\circ) & \sin(120^\circ) \\ 0.5 & 0.5 & 0.5 \end{bmatrix} \begin{bmatrix} Sa \\ Sb \\ Sc \end{bmatrix} \quad (2.31)$$

As before, the variable S represents voltage, current or flux linkage and the zero sequence component  $S\sigma$  is always zero for a balanced 3 phase system. The second transformation simply converts from stationary  $\alpha$ - $\beta$  frame to the rotating d-q frame as

$$\begin{bmatrix} Sq \\ Sd \\ S\sigma \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} S\alpha \\ S\beta \\ S\sigma \end{bmatrix} \quad (2.32)$$

It can be easily verified that multiplication of the above two transformations lead to Eq. 2.18. Since the d-q frame itself is rotating at a synchronous frequency, all sinusoidally modulated terms vanish after the transformation into this d-q frame. Notice that since the transformation of Eq. 2.18 (and Eq. 2.31) is not unitary (A square matrix is unitary if its inverse is the same as its transpose), the power and torque of the 2-phase equivalent system is different from those of the original 3-phase system. To calculate power and torque from the 2-phase equivalent circuit, (3/2) factor has to be included as shown in Eq. 2.28-2.30. The reason that this non-unitary transformation of Eq. 2.18 is popular is because the magnitude of voltages, currents and flux linkages are the same in both frames. See Appendix for further information regarding a unitary transformation. In this 2-phase equivalent circuit, inductances are roughly (3/2) times those of the actual 3 phase value (refer to Eq. 2.24-25). These "synchronous inductances"  $L_d$  and  $L_q$  are the effective inductances seen by a phase winding during balanced operation. In addition to the self-flux linkage of one phase, additional flux linkages are produced from the other two phase currents.

For some applications, it is useful to define voltage vector  $\mathbf{Vs}$  and current vector  $\mathbf{Is}$  whose magnitudes are

$$|\mathbf{Vs}| = V_s = \sqrt{V_q^2 + V_d^2}, \quad (2.33)$$

$$|\mathbf{Is}| = I_s = \sqrt{I_q^2 + I_d^2}. \quad (2.34)$$

Assume that current vector  $\mathbf{Is}$  is  $\theta_m$  degrees ahead of the q-axis. Then, the relation between the stator current magnitude  $I_s$ , and  $I_d$  and  $I_q$  are

$$I_q = I_s \cos \theta_m, \quad (2.35)$$

$$I_d = -I_s \sin \theta_m, \quad (2.36)$$

and Eq. 2.30 can be expressed in terms of  $\theta_m$  as

$$T = (3/2)(P/2)(\lambda_m I_s \cos \theta_m + 0.5 (L_q - L_d) I_s^2 \sin 2\theta_m). \quad (2.37)$$

For surface PM motors whose  $L_q = L_d$ , the reluctance torque term of the above equation vanishes and the above equation is reduced to

$$T = (3/2) (P/2) \lambda_m I_s \cos \theta_m. \quad (2.38)$$

Here, the maximum torque is produced when  $\theta_m = 0^\circ$ , or the angle between the stator flux linkage vector and the PM flux linkage vector on the rotor is 90 degrees which is analogous to the characteristics of a separately excited DC motor. For interior PM synchronous motors, the reluctance torque is not negligible and higher torque can be produced by appropriately adjusting the angle  $\theta_m$ .

### III. CHARACTERIZATION OF MODEL PARAMETERS

In order to establish a 2-phase equivalent circuit of a given motor, there are four motor parameters that are to be determined. These are stator resistance  $R_s$ , inductances  $L_q$  and  $L_d$ , and PM flux linkage  $\lambda_m$ . This portion is adapted from [7].

### (1) Stator resistance $R_s$

Since  $R_s$  is the resistance between line-to-neutral,  $R_s$  would be one half of the measured line-to-line resistance. For most small to medium size PM synchronous motors, skin effect is not significant and can be safely neglected. Winding resistance value is highly temperature-dependent. When winding resistance  $R_o$  is measured, temperature  $T_o$  ( $^{\circ}\text{C}$ ) of the winding must be recorded and the resistance  $R_t$  at another temperature  $T$  should be calculated using the following formula.

$$R_t = R_o (K + T) / (K + T_o) \quad (3.1)$$

where  $K$  is the constant determined by the material ( $K=234.5$  for copper). Resistance value at  $25^{\circ}\text{C}$  is often used for published data.

### (2) Synchronous Inductances $L_q$ and $L_d$

To measure inductance, digital RLC meter or inductance bridge is often used. These measurements give inductance value at a small current and may not be suitable to predict characteristics of the motor. The small signal inductance value may be lower than unsaturated inductance value since permeability of iron decreases at very low flux densities [8]. Since inductance is also subject to saturation, we need to measure inductances at various current level of interest. Furthermore, when motor windings are delta-connected or Y connected without neutral line for external access, one can only access line-to-line value. Even when phase inductance value is measured, calculating synchronous inductances based on Eq. 2.24 and 2.25 may not be accurate because we do not know the leakage inductance. In the following, a novel method of directly measuring synchronous inductances  $L_q$  and  $L_d$  with biased current is illustrated.

In order to measure synchronous inductance, we must maintain balanced three phase current condition. Connection diagram of Fig. 3.1 maintains balanced 3 phase current condition at  $\theta = 0^{\circ}$ . When the rotor q-axis (d-axis) is aligned with the center of a-phase winding,  $L_q$  ( $L_d$ ) can be derived from the measured equivalent inductance  $L$  of the circuit in Fig. 3.1, depending on the rotor angle.

$$L_q = (2/3) L(\theta = 0^{\circ}), \quad L_d = (2/3) L(\theta = 90^{\circ}) \quad (3.2)$$

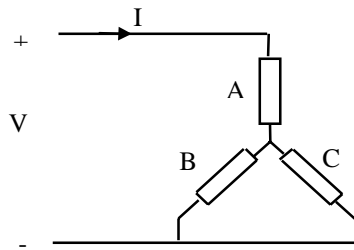


Fig. 3.1 Connection Diagram to measure Synchronous Inductance

This relation can either be proven in a straightforward manner by using Eq. 2.1-2.15, or intuitively as follows. When the rotor is locked at 0 degree (q-axis = a-axis) and supply voltage  $V$  is applied as in Fig. 3.1, Eq. 2.26 becomes

$$V = \{(3/2) R_s + (3/2) L_q p\} I_q \quad (3.3)$$

since  $V_q = (2/3)V$  and  $V_d = 0$ . Since a-phase current  $I$  is the same as  $I_q$ , and the equivalent resistance of the circuit of Fig. 3.1 is  $(3/2) R_s$ , the equivalent inductance seen from the supply source is  $(3/2) L_q$ . Similar explanation can also be applied to  $L_d$  when the rotor is locked at  $90^{\circ}$ .

NEMA [9] suggested two methods of measuring inductance. One is called “biased inductance bridge method”. It describes connection diagram when traditional inductance bridge is used. Recently, new digital RLC meters are available. Detailed procedures and circuits to measure inductances with a biased current on these meters differ by manufacturer. Often, special optional devices in addition to the meter unit is necessary. This method, if available, offers a simple and accurate measurement. When the biased inductance measurement is unavailable,

inductance can also be measured from the time response as illustrated in Fig. 3.2. It is called the “current decay method.”

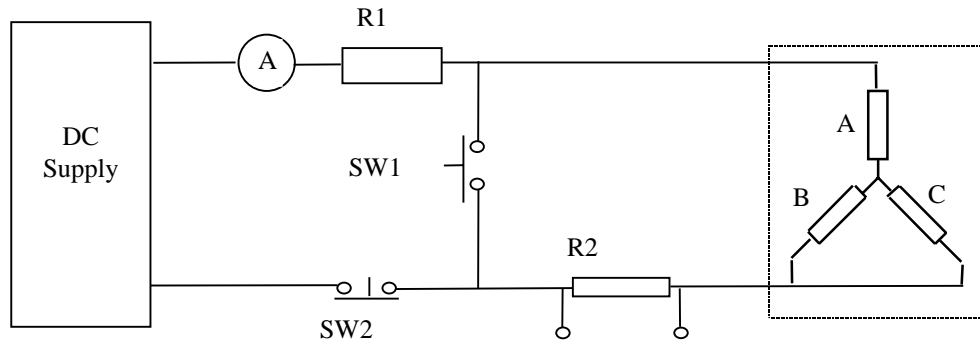


Fig. 3.2 Current Decay Method

In the above figure, R1 is to limit current to a desired level and R2 is a shunt resistance for oscilloscope measurement. Switches SW1 (normally open) and SW2 (normally closed) are tied together so that SW1 should make before SW2 breaks. Measurement procedure is as follows.

- (1) Adjust supply voltage or resistance R1 so that current is at desired level  $I_{init}$ . Allow time for current to settle. Note that when current decay method is used,  $I_{init}$  should be around 150% - 200% of the desired current level.
- (2) While observing oscilloscope via shunt resistor R2, close SW1 and then open SW2. Capture waveform of current decay and measure the duration  $T_d$  from the starting time of current decay to the time the current reached to 37% of  $I_{init}$  (63% of total change).
- (3) Measure the equivalent resistance  $R$  of the motor accurately before winding temperature changes.
- (4) Calculate inductance value by using the following formula.

$$L = T_d R \tag{3.4}$$

Although the method is very simple and does not require special measurement device, this method is not recommended for accurate measurement.

A complete inductance measurement at various current magnitude and angle can be performed with specially built motors with all 6 winding leads accessible. Fig. 3.3 illustrates the method applicable to Y connected motors. Let  $a'$ ,  $b'$  and  $c'$  leads denote neutral leads. For the test,  $b'$  and  $c'$  leads are connected together. Let T1 be the measurement terminal which may also be biased, while T2 (bias terminal) is strictly for current bias only. Throughout the test, variable resistor should be adjusted so that balanced 3 phase current condition ( $I_a + I_b + I_c = 0$ ) should be maintained.

Q-axis inductance ( $L_q$ ) at various conditions can be measured as follows.

- (1) Align rotor q-axis into a-axis by energizing current into T2 terminal ( $c(+)$  to  $b(-)$  with a phase float) and lock the shaft. Rewire the circuit as Fig. 3.3 and connect a DC current supply to T2 while biased inductance measurement circuit is connected to T1.

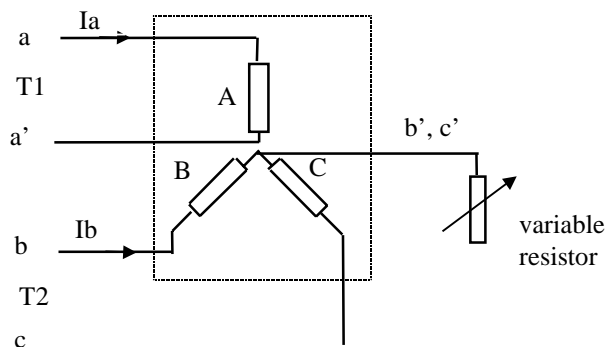




Fig. 3.3 Circuit diagram for General Inductance Measurement

(2) To measure  $L_q$  at the current magnitude  $I_s$  and angle advance  $\theta$  (relative to q-axis), adjust supply current and variable resistor so that

$$\begin{aligned} I_a &= I_s \cos \theta \\ I_b &= I_s \cos (\theta - 120^\circ) \\ I_c &= I_s \cos (\theta + 120^\circ) \end{aligned}$$

and measure inductance.  $L_q$  would be  $(3/2)$  times the measured value.

(3) Repeat the above step for various  $I_s$  and  $\theta$  and plot the data

For d-axis inductance ( $L_d$ ) at various conditions can be measured as follows.

(1) Align rotor d-axis into a-axis by energizing current into T2 terminal (a(+)) to b and c(-) with neutral leads commoned) and lock the shaft. Rewire the circuit as Fig. 3.3 and connect a DC current supply to T2 while biased inductance measurement circuit is connected to T1.

(2) To measure  $L_d$  at the current magnitude  $I_s$  and angle advance  $\theta$  (relative to q-axis), adjust supply current and variable resistor so that

$$\begin{aligned} I_a &= I_s \cos (\theta + 90^\circ) \\ I_b &= I_s \cos (\theta - 120^\circ + 90^\circ) \\ I_c &= I_s \cos (\theta + 120^\circ + 90^\circ) \end{aligned}$$

and measure inductance.  $L_d$  would be  $(3/2)$  times the measured value.

(3) Repeat the above step for various  $I_s$  and  $\theta$  and plot the data

### (3) PM Flux Linkage $\lambda_m$

The flux linkage of the permanent magnet,  $\lambda_m$  can be obtained by measuring the no-load line-to-line rms voltage  $V_{nl}$  of the motor while it is driven through the shaft at a constant speed of  $\omega_m$ . From Eq. 2.26,

$$\lambda_m = \sqrt{(2/3)} V_{nl} / \omega_m \quad (3.5)$$

where  $\omega = (P/2) \omega_m$ . This expression is valid when  $I_q$  and  $I_d$  are negligibly small. It can also be calculated from the manufacturer's data sheet. Since most motor data sheet specify its back-emf constant  $K_b = V_{nl} / \omega_m$ ,

$$\lambda_m = \sqrt{(2/3)} (2/P) K_b \quad (3.6)$$

Another method of obtaining  $\lambda_m$  is from the measurement of torque and stator current while maintaining orthogonal relationship between rotor and stator magnetic fluxes ( $I_d = 0$ ). Measurement can be done at standstill condition while orthogonality is assured.  $\lambda_m$  can be calculated from

$$\lambda_m = (2/3) (2/P) T / I_s \quad (3.7)$$

Note that the peak current value (not rms value) has to be used for  $I_s$  in the above equation.. It is convenient to predict armature reaction by plotting measured  $\lambda_m$  values at various current level.

In order to characterize saturation and armature reaction vs current, Frolich's formula may often be used. Assume that  $L_{q0}$ ,  $L_{d0}$  and  $\lambda_{m0}$  be values at linear region where magnitude of  $I_q$  is smaller than  $I_0$ . Since  $L_q$  is subject to saturation and  $L_d$  and  $\lambda_m$  are subject to armature reaction at high currents, the following expressions are often used when magnitude of  $I_q$  is greater than  $I_0$ .

$$L_q(I) = L_{q0} (a + I_o) / (a + |I_q|) \quad (3.8)$$

$$L_d(I) = L_{d0} (b + I_o) / (b + |I_q|) \quad (3.9)$$

$$\lambda_m(I) = \lambda_{m0} (b + I_o) / (b + |I_q|) \quad (3.10)$$

### EXAMPLE

A Kollmorgen 6 pole PM synchronous motor is measured as follows.

$$R_{l-1} = 1.90 \Omega, @ 25^\circ\text{C}$$

$$L(0^\circ) = 21.15 \text{ mH up to } 10\text{Arms}, L(0^\circ) = 16.08 \text{ mH at } 20\text{Arms}$$

$$L(90^\circ) = 12.20 \text{ mH up to } 10\text{Arms}, L(90^\circ) = 10.73 \text{ mH at } 20\text{Arms}$$

$$V_{nl} = 106.8 \text{ V @ } 1000\text{RPM},$$

$$\text{Orthogonal torque} = 17.6 \text{ Nm at } 10 \text{ Arms}, 31.0 \text{ Nm at } 20\text{Arms}$$

Now,  $R_s$  is

$$R_s = 1.90 / 2 = 0.95 \Omega$$

$\lambda_m$  according to Eq. 3.6 is

$$\lambda_m = 0.816 (2/6) 106.8 / (1000 2\pi / 60) = 0.277 \text{ Wb-T}$$

And according to Eq. 3.7,

$$\lambda_m = (2/3) (2/6) 17.6 / (1.4142 10) = 0.2765 \text{ Wb-T.}$$

For linear region up to 10Arms,

$$L_{q0} = 14.10 \text{ mH}, L_{d0} = 8.13 \text{ mH}, \lambda_{m0} = 0.277 \text{ Wb-T}$$

For saturated values, substituting measurement values in Eqs. 3.8-3.10,

$$L_q(I) = 14.10 (21.3 + 10) / (21.3 + |I_q|)$$

$$L_d(I) = 8.13 (63.3 + 10) / (63.3 + |I_q|)$$

$$\lambda_m(I) = 0.277 (63.3 + 10) / (63.3 + |I_q|)$$

### APPENDIX: UNITARY FRAME TRANSFORMATION

Sometimes, the following transformation is used instead of Eq. 2.18-2.19. For 3 phase quantities  $S_a, S_b$  and  $D_c$ , the transformation in matrix form is given by

$$\begin{bmatrix} S_q \\ S_d \\ S_o \end{bmatrix} = \frac{1}{\sqrt{2/3}} \begin{bmatrix} \cos \theta & \cos(\theta - 120) & \cos(\theta + 120) \\ \sin \theta & \sin(\theta - 120) & \sin(\theta + 120) \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} S_a \\ S_b \\ S_c \end{bmatrix} \quad (A.1)$$

Also, its inverse transformation exists and is

$$\begin{bmatrix} S_a \\ S_b \\ S_c \end{bmatrix} = \frac{1}{\sqrt{2/3}} \begin{bmatrix} \cos \theta & \sin \theta & 1/\sqrt{2} \\ \cos(\theta - 120) & \sin(\theta - 120) & 1/\sqrt{2} \\ \cos(\theta + 120) & \sin(\theta + 120) & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} S_q \\ S_d \\ S_o \end{bmatrix} \quad (A.2)$$

The transformation is unitary and the power is preserved on both systems. With this transformation,  $3/2$  terms on both Eq. 2.28 and Eq. 2.30 are not necessary. In the 2-phase equivalent circuit, vector variables in the d-q- axis have  $\sqrt{2/3}$  times the magnitude of the original 3-phase vectors. In other words, an equivalent 2-phase system results if

- (1) design a 2 phase system with inductance values of  $L_d$  and  $L_q$  as in Eq. 2.24-2.25.
- (2) permanent magnet flux has to be  $\sqrt{3/2}$  times that of 3-phase system
- (3) apply  $\sqrt{3/2}$  times the voltage and current magnitudes as compared to its 3-phase quantities.

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